Departmental Exam 2007-2008

This year’s test does not separate the short and long questions into different sections. It would be a good idea to skim the entire test before beginning so that you can budget your time sensibly. You have 3 ½ hours, plus an extra half hour if you want. You can either write your answers on the test itself or on separate sheets of paper. Let us know if you need a bathroom break, but please remember that the exam is closed book.

You should try to answer all the questions. Credit will be given for partial answers, but not for material irrelevant to the question. Brevity is appreciated, but not to the point of obscurity. Correct answers matter, but so does evidence that you know what you are doing. If you get answers that are dimensionally wrong, or clearly off by orders of magnitude, show that you can see this, even if you cannot fix the problem.

(1)

a. What is the most abundant mineral in Earth's upper mantle?

b. What is the average density of Earth's crust?

c. What is the mean ocean depth? Deepest depth?

d. What is the P-wave velocity in ocean water? S-wave velocity?

e. My GPS instrument collects 100 independent measurements of height, from which I compute a mean of 700 m and a standard deviation of 50 cm. What is the standard error in the mean?

f. I test my GPS data against a normal distribution using a Kolmogorov-Smirnov (K–S) test and get a probability of 0.90. What does this mean?

g. At what depth is the upper mantle/lower mantle boundary?

h. What range of age is appropriate for Carbon-14 dating? Potassium-Argon?

i. How strong is Earth's magnetic field? Mars’?
j. Would you characterize Earth's geomagnetic field spectrum as red, white, or blue?

k. What would be a typical surface heat flow? Geothermal gradient?

l. What is the typical thickness of oceanic crust? Continental crust?

m. Give a definition of earthquake b-value.

n. Define scalar seismic moment $M_0$

o. Sort from fastest to slowest: Love waves, Rayleigh waves, P waves, S waves

p. Draw a sketch defining strike, dip and rake for an earthquake fault.
(2) Why does ocean depth vary approximately as one over the square root of seafloor age?

(3) What are the three main types of plate boundaries and what type of earthquake focal mechanism typically occurs on each?

(4) Sketch a hypsometric curve for the earth (i.e., fractional area versus elevation and depth). Label the elevation/depth axis but don’t worry about an absolute scale on the area axis. Label and describe the main features of this histogram.

(5) Assume the lithosphere of Venus has evolved to a steady-state temperature profile. Given a current heat flow of $4 \times 10^2$ W m$^{-2}$, a surface temperature of 450°C, a mantle temperature of 1500°C and a thermal conductivity of 3.3 W m$^{-1}$ C$^{-1}$, calculate the thickness of the lithosphere.
(6) The decay constant for radioactive decay of $^{87}$Rb to $^{87}$Sr is $1.42 \times 10^{-11}$ yr$^{-1}$.

(a) Set up the differential equation for the rate of change in $^{87}$Rb atoms versus the current number of $^{87}$Rb atoms.

(b) Solve the differential equation for the number of atoms as a function of time assuming an initial number of $N_0$.

(c) What is the half life?

(d) If the Earth started with $N_0$ atoms of $^{87}$Rb, how many are remaining today?

(7) Abyssal hills on the seafloor have a characteristic wavelength of 10 km and produce a gravity anomaly amplitude of 5 mGal at the bottom of the ocean. What is the amplitude of the gravity anomaly at the sea surface where the mean ocean depth is 3 km?
(8) (a) What is the orbital radius for a satellite in geostationary orbit (i.e. orbit period = 1 day)?
Note that: \( G = 6.67 \times 10^{-11} \text{ m}^2 \text{ kg}^{-1} \text{ s}^{-2} \) \( M_e = 5.98 \times 10^{24} \text{ kg} \)

(b) \( J_2 \) is about \( 10^{-3} \). At the surface of the Earth the gravitational effect of the \( J_2 \) term is about \( 10^{-3} \) times the total acceleration of gravity. What is this ratio for the satellite in geostationary orbit?

(9) For inaconformal map projections (such as the Mercator) the scale is not the same everywhere (which is why Greenland looks so big), but shapes are preserved in any very small region. What kind of strain does this correspond to? Which tensor components, or combinations of components, are zero in this case? What about an equal-area map projection, in which the scale in each small region is the same, but shapes are distorted?

(10) Suppose we have a planet made of incompressible material of density \( \rho \). Write down and solve the equations to give \( g \) and the pressure as a function of radius; take the radius of the planet to be \( R \). What is the Poisson’s ratio of the material? Would you expect there to be any kinds of seismic waves that could propagate, and if so what would they be?
(11) The above figure shows data relating earthquake magnitude to fault rupture length. From these data, how would you go about finding the most likely value of rupture length for a given magnitude? How about the other way around (i.e., finding the most likely magnitude for a given rupture length)? What kind of estimator is involved, and what assumptions make it valid (if it is)?

(12) The local Gas & Electric company guarantees that all its power is produced in the frequency band $60 \pm 0.03$ Hz. You want to estimate the power spectrum of the local vertical geomagnetic field $Z$ between periods of roughly one minute and thirty minutes. Your grant contains no money for an analog attenuator of the 60 Hz signal, so you decide to measure $Z$ every $x$ seconds, choosing $x$ so that the power-line signal will not be aliased into the lowest third of your Nyquist band. That is, if $f_N$ is the Nyquist frequency, no power company frequencies alias into frequencies $f$ satisfying $f_N/3 < f < f_N/3$. What is the largest value you can choose for $x$, and if you choose that value, what is the shortest period at which you can estimate the spectrum of $Z$?
(13) (a) Explain the term isostasy.

(b) What is meant by local isostatic compensation? Explain the difference between the Pratt and Airy mechanisms.

(c) What is regional compensation? What property of the lithosphere can be estimated from a regional compensation model if appropriate quantities are measured? What are those quantities?

(d) Which of the two, local or regional compensation, is at work at mid-ocean rises?

(e) A broad mountain range is locally compensated. What is the effect of the range on the free-air gravity anomaly? What is the effect on the geoid height? Explain your answer.
(14) (a) State the induction equation governing the evolution of the magnetic field in a moving, electrically conducting fluid.

(b) The magnetic Reynolds number $R_m$ gives the ratio of the magnitudes of two terms in the equation. What processes do the terms represent? Derive the usual order of magnitude expression for $R_m$ based on typical parameters of the system.

(c) Make an estimate of the value of $R_m$ for fluid motions in the Earth’s core? You will need to assume values for some properties of the core. What consequences for the study of the magnetic secular variation follow from the magnitude of $R_m$?
(15) Gauss showed that the geomagnetic field could be represented by the gradient of a scalar potential $\psi$ with spherical harmonics composed of two distinct parts, an interior part and an external part.

(a) Write down this spherical harmonic expansion and explain what the symbols mean. What are the major physical sources of the internal part of the field, and which group of spherical harmonic coefficients is the largest? What are the major physical sources of the exterior field? In addition to magnitude, what other important characteristic distinguishes the exterior field from the interior one? What is the spherical harmonic degree of the terms that dominate in the exterior field?

(b) The gravitational potential is also derived from a scalar potential, $V$. It too has a spherical harmonic expansion, with internal and external parts. Write out the expansion. Which terms are present here, but not in the magnetic potential and why? What causes the exterior part? Which spherical harmonic terms are the largest in the exterior part?
The topography $h$, defined by elevation above sea level, in a region can be modeled by an isotropic stochastic process with Power Spectral Density (PSD) given by

$$S_h(k) = \frac{a}{2(b^2 + k^2)^{3/2}}$$

where $a$ and $b$ are constants, and $k$ is the horizontal wavenumber magnitude.

How would you find the root-mean-square (RMS) deviation from the mean value, based on the PSD, $P_h$?

Show how to calculate $P_h(k_z)$, the PSD of elevation along a straight road across the region. There is no need to actually perform the detailed calculations.

How is the autocovariance of the topography on the road found from the PSD?
The autocovariance of the topography along the road can be shown to be

$$R_h(x) = \frac{\pi a}{b} \exp(-2\pi bx), \quad x \geq 0$$

(2)

If $a = 0.2$ m, and $b = 0.0005$ m$^{-1}$, what is the RMS deviation from the mean elevation?

With the same constants, calculate the RMS elevation difference between points 100 m apart on the road. Would you regard this road as very hilly, or just smoothly undulating? Explain your answer.