**Geophysics Departmental Exam: 2004**

**Part 1**

This section is 90 minutes, closed book, and consists of questions designed to test your knowledge of facts and figures in the geosciences. The focus will be on the five geophysics core courses, with additional questions on other relevant areas. You are expected to answer all questions. It is better to be brief and to the point than to try to write a long answer, and better to admit ignorance than to be in error.

1. **What is the evidence that much of this Earth’s core is fluid, and the rest solid?** About how big are these two parts? What are reasonable values for the Poisson’s ratio of each part?
2. **How is a random variable described?**
3. **Suppose**
   \[ A = \begin{bmatrix} 3 & 4 & 1 \\ 0 & 2 & 3 \\ 5 & 7 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \]
   Then, assuming the summation convention, what are \( A_{ii} \) and \( A_{ji} b_j \)?
4. **Give two examples of seismic low velocity zones in the Earth.**
5. **What is the ray parameter of a seismic wave?**
6. **What is the principle of isostasy?**
7. **Why are elements with periodic number less than and equal to that of iron much more abundant than elements with higher periodic number?**
8. **Derive formulas for the acceleration of gravity and pressure inside a constant-density planet. Assume hydrostatic equilibrium and zero surface pressure.**
9. **What is the average thickness of (a) the continental and (b) oceanic Earth’s crust?** What are the thickest and thinnest values for each? Give an example of where these extremes occur.
10. **Describe, briefly, earthquake magnitude and moment in terms of data that would be used to estimate them. How are they related?**
11. **How many material constants are needed to completely characterize static deformation of an isotropic elastic solid?**
12. **What is a FIR filter?** Give one thing it can be used for.
13. **Estimate the maximum shear stress that can be supported by crustal rocks at depth of 10 km (order of magnitude).**
14. **What is the expected value of a random variable?** What do we mean when we say that a statistical estimator is unbiased? What do we mean when we say that a statistical estimator is robust?
15. **What is the difference between the Earth’s crust and the lithosphere?**
16. **What is convolution?** What is the convolution theorem?
17. **What is a triple junction?** What is the only type of one that is stable?
18. **Calculate the Fourier transforms of the following functions:**
    
    (a) \( \text{box}(x) \equiv \begin{cases} 1 & |x| \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \)
    
    (b) \( \text{sinc}(x) \equiv \frac{\sin \pi x}{\pi x} \)
19. Explain the meaning of the following terms: (a) geopotential; (b) reference ellipsoid; (c) geoid; (d) geoid height. Give an approximate relationship between geoid height and the geopotential on the reference ellipsoid. Estimate the error in this formula.

![Figure 20](image-url)

**Figure 20**

20. Figure 20 shows a Q-Q plot of some data (NS distance between two GPS points). How was this produced? What does its shape tell you about the distribution of the data, and what does that distribution tell you about how best to estimate the “true” value of the data?

21. What is the Earth’s flattening? Why does it occur?

22. What are the main types of plate boundary? Give one geographically specific example of each type. What class of earthquake focal mechanism predominates on each type?

23. Suppose there is a large earthquake in a viscoelastic half space where the viscosity is $10^{21}$ Pa·s and the shear modulus is $6 \times 10^{10}$ Pa. About how long will it take for the stress from the earthquake to decay to $1/e$ of the value just after the earthquake?

24. Define the Lowes spectrum of the geomagnetic field (sometimes also referred to as the Lowes-Mauersberger spectrum). Give a physical interpretation. From what kinds of measurements is it derived, and how is it calculated?

25. Why is it permissible to downward continue the geomagnetic field down to the core, but no further, and what are the limitations of this calculation? Why is it not permissible to do the same thing for the Earth’s gravitational field?

26. A time series of ocean wave height would be in meters. What would the units (or dimensions) be for the power spectral density of these data?

27. Most of the energy in ocean swell occurs at periods of 5-20 seconds. How rapidly should we sample records of swell to make a digital series?

28. Explain what is meant by an adiabatic temperature gradient. How does this relate to convection in a fluid?

29. Write down a constitutive relationship between tensor stress and strain for a Maxwell (serial spring and dashpot), and derive an expression for the Maxwell relaxation time (assume no temporal variations in strain).

8 July 2004
30. What is the difference between basalt and granite? Between granite and rhyolite? What proportions of the bedrock of the Earth’s surface is composed of each of these rock types? A rough but reasonable guess is fine.

31. Suppose a seafloor spreading rate is given as 30±3 mm/yr. What does the 3 mean? What does it mean to say that the 95% confidence limits for the rate are 25 and 35 mm/yr?

32. Define, briefly, what elastic, plastic, and viscous behavior are for materials. Give an example for the Earth of each kind of behavior.

**Part 2**

This section is two hours, open book, and consists of problems that will require some amount of reasoning and equation solving. Answer **at least 8 problems** of your choice. Even if you cannot solve the equations, you will get credit for posing the problem correctly, and describing a path for getting a solution. This will test your ability to approach a research issue. While this portion is “open book”, it may not be an advantage to rely on this: books can be very distracting and time-wasting. It is better to demonstrate an ability to think the problem through, perhaps from first principles. Remember that use of equations is often better (and briefer) than lots of words; but the equations must be appropriately explained to show why you have written them.

---

**Figure A**

The varying rotation rate of the Earth is reported as a time series of the change in the length of the day, \( \Delta T(t) \). A century of such data were analyzed and the (one-sided) PSD estimated, a simplified version of which appears in the Figure.

(a) Give an interpretation of the power spectrum, and based upon it calculate the variance of the LOD record, and the standard error of the clocks used to measure \( \Delta T \). If the highest frequency in the graph is the Nyquist frequency, how many measurements were used to calculate the PSD? Estimate numerical quantities by eye from the figure.

(b) The day is slowly increasing in length at a rate of about 2 milliseconds per century. Would you expect to be able to detect this signal in the record that was used to generate the PSD? Why is simple least-squares estimation a poor way to estimate the secular change in this case?

(c) From the given PSD of length of day changes find the power spectra of the Earth’s angular velocity \( \omega \), and of the angular acceleration \( \frac{d\omega}{dt} \).

---

B. One frequently reads about such things as the gravity field of spherical harmonic degree 4, meaning the field that results when only the degree 4 spherical harmonics are summed, while the rest are dropped. Since the spherical harmonic coefficients all vary with the orientation of the reference axes, such “degree-n” functions are seemingly coordinate dependent.
Write the degree-$n$ function on the surface of the unit sphere as an integral of the original function, and use this result to show that a degree-$n$ function does not depend on the choice of axes.

Figure C

Figure C shows the displacement for a harmonic plane wave at $t = 0$, traveling in the $x$ direction at 5 km/s. Write down an equation for this wave that describes displacement, $u$, as a function of $x$ and $t$. What is the maximum strain, and acceleration for this wave? Suppose it travels 1000 km through a medium with $Q$ of 100; what will its amplitude then be?

D. Within a spherical shell the magnetic field $B$ can be written as the sum of two parts: $B = S + T$, where $S$ is called the poloidal part, and $T$ is the toroidal part. Define these terms. What are the distinguishing features of the toroidal part? In which of the following regions would you expect the magnetic field to possess a significant toroidal part: (a) The outer core; (b) the upper mantle; (c) the Ocean; (d) the atmosphere below 50 km; (e) the ionosphere. Explain your answers.

E. A downgoing P wave in a medium with a P velocity of 6 km/s travels through a “corner” shaped structure as shown, with incident and exiting angles as shown in Figure E. Write a formula relating the various angles and the velocities. What is the P velocity within the corner shaped medium? If the angle of incidence is 45°, what is the possible range of angles for the outgoing wave, given an arbitrary velocity contrast.

F. Given the rotation pole between the African and South American plates (pole; latitude = 62.5°, longitude = 320.6°, rate = $5.58 \times 10^{-9}$ radian/yr), calculate the spreading rate at a point on the northern Mid-Atlantic Ridge (lat = 30°, long = 319°). Use vector methods, not spherical trigonometry.

G. Assume the lithosphere of Venus has evolved to a steady-state temperature profile. Given a current heat flow of $4 \times 10^{-12}$ W m$^{-2}$, a surface temperature of 450°C, a mantle temperature of 1500°C and a thermal conductivity of 3.3 W m$^{-1}$ C$^{-1}$ calculate the thickness of the lithosphere.

H. Abyssal hills, which form at seafloor spreading ridges, have a characteristic spacing of 4 km and have peak-to-trough amplitude of 400 m. Assume they are infinitely long in the ridge-parallel direction. What is the amplitude of their gravity anomaly at the surface of the
ocean? What is the amplitude at the altitude of a satellite (400 km)?

I. Stress measurements in a borehole have revealed a north-sourth compression $T_{NN}$, an east-west compression $T_{EE}$, and a shear stress $T_{NE}$ on a vertical plane striking east-west. Assuming that the stress state is two-dimensional, find the magnitude of the principal stresses.

J. Determine the total heat output on the creeping section of the San Andreas Fault in Watts. Assume the shear stress varies with depth as $\tau = f \rho g z$, with a coefficient of friction ($f$) of 0.6. The fault is 25 km deep, 100 km long, and creeping at 35 mm/yr. How would you find the profile of heat flow at the ground surface? What assumptions could you make to simplify finding the solution? Be specific and use equations.

K. For the postglacial rebound problem, suppose we have an ice load whose depth is described by $h(x_1, x_2, t)$: that is, it is a function of position and time (making the Earth flat). What would be the equations needed to solve for the deformation of a uniform, self-gravitating Earth under such a load, assuming that the Earth is elastic? How does this change if it is Maxwellian? Most data on rebound are measurements of sea level, not absolute displacement. Why would these differ, and how would you include this difference in the equations?