IGPP

Departmental Examination

2002 (required)
Departmental Exam: June 2002

Please spend 15 minutes reading the exam before entering the start time.

start time:
end time:

This is a closed-book 3 1/2 hour exam. You may use a calculator but do not consult any notes, books, or colleagues for help. The exam has two sections: geophysics and other. Correct answers matter but so does evidence that you know what you are doing. If you get results that are dimensionally wrong, or clearly off by orders of magnitude, show that you see the problem even if you cannot fix the problem.

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Departmental Exam: Geophysics Section – June 2001

ATTEMPT ALL QUESTIONS IN THIS SECTION

1. (a) Evaluate the Fourier transform of the boxcar function

   \[ f(x) = \begin{cases} 1 & |x| < 1/2 \\ 0 & |x| > 1/2 \end{cases} \]

   where

   \[ F(k) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i k x} dx. \]

   (b) Plot the result. What happens at \( k \) approaches zero and \( k \) approaches infinity.

2. (a) Briefly explain the processes of upward and downward continuation of a harmonic function. Explain how this is done when you are given a spherical harmonic expansion of the Earth’s geomagnetic field.

   (b) The observed geomagnetic field at the Earth’s surface is commonly downward continued to the surface of the core, but there is a fundamental restriction on the resolvable length scales there. Why? What is smallest resolvable scale? Why is it possible to downward continue the magnetic field to the core, but not possible for the Earth’s gravity field?

   (c) A lineated magnetic anomaly at the surface of the ocean is found to have a wavelength of about 100 km and an amplitude of 500 nT. What is the approximate amplitude of the anomaly on the seafloor, 4 km below, and what is its amplitude at the altitude of Magsat, 400 km above the surface?

3. Consider the low-order spherical harmonic expansion for the Earth’s gravitational potential, \( V \). For each term in the list below, draw a sketch of the shape of the corresponding geoid perturbation from a sphere.

   (a) Degree zero: \( Y_0^0 \). What property of the Earth does this harmonic measure?
(b) Degree one: \( Y_1^m \). The coefficients of these spherical harmonics are all exactly zero. Why?

(c) Degree two: \( Y_2^o \). The (fully normalized) coefficient of this harmonic is nearly 300 times larger than any other besides that for \( l = 0 \). Explain why.

(d) Degree two. \( Y_2^{-1} \), \( Y_2^1 \). These two coefficients are almost exactly zero, and too small to measure. Why are they small?

(e) Degree two: \( Y_2^{-2} \), \( Y_2^2 \). Sketch the shape of the geoid in the equatorial plane for which these terms are responsible. Draw the two principal axes of inertia, A and B on your sketch.

4 (a) Define the autocorrelation function for a stochastic process.

(b) What is the relationship between autocorrelation and power spectral density?

(c) Sketch the autocorrelation functions and power spectra you would expect to be associated with the following time series (5000 data points). Don't forget to label the axes. What is the approximate value for the peak in the autocorrelation function.
(d) What is the most appropriate computer language(s) for the following task?
   i) You have a data series of 10,000 samples and need to compute mean, variance, trends and spectra.
   ii) You are managing a geophysical network delivering 20 Hz data from 3-channel sensors at 15 stations. You wish to publish hourly means and variances on the web.

5 A vertically propagating upgoing plane wave is incident on a sediment-basement interface where the seismic velocity drops from 4 km/s to 2 km/s. A cylindrically shaped depression (with radius of curvature $r = 1$ km) focuses (approximately) the ray paths to a focus point within the sediments. Solve for the height of the focus point above the lowest point of the interface.
6 Assume there is a mantle plume directly under the island of Hawaii. As the Pacific plate passes over plume, the temperature at the base of the plate is suddenly increased.

(a) Given the plate thickness of 60 km and a thermal diffusivity \( \kappa = 10^{-6} \text{ m}^2 \text{s}^{-1} \), approximately how long it will take for the heat flow anomaly to appear on the surface.

(b) The velocity of the Pacific plate relative to the plume is 100 mm/yr. Design (sketch) an experiment to detect the anomalous heat flow caused by the plume. Where would you go and what would you do? Do you see any problems performing this experiment?

7 Determine the value of the following integral using the method of complex contour integration.

\[
\int_0^\infty \frac{x \sin(x)}{1+x^4} \, dx
\]

(a) What is an appropriate integral to consider in the complex plane?

(b) Choose a suitable contour and explain the reason(s) for your choice.

(c) Use the Cauchy Residue Theorem to evaluate the contour integral and indicate the value that results for the original integral.

(Hint: What are some of the important properties of the numerator and denominator?)

8 You are given a set of temperature measurements \( T_i \) at depths \( z_i, i=1, \ldots, N \), in a borehole and you want to estimate the temperature gradient. Assume a linear variation in temperature with depth and initially assume there are no errors in the depth measurements. The temperature measurements have errors with a constant standard deviation.

(a) Derive the least-squares solution for the model parameters.

(b) How would you modify this approach if the standard deviations in the temperature measurements were not all the same value? (How does this change \( \chi^2 \)?)

(c) Assume now that there are no errors in the temperature measurements but that there are errors in the depth measurements. How would you solve for the model parameters in this case? Would you get the same answer for the temperature gradient as you estimated from part (a)?
9 You make broad-band recordings of body waves from an earthquake at a variety of distances from the source along a great-circle path. The ratio of short period energy to long period energy decreases with increasing epicentral distance - why?

10 For an isotropic elastic solid, the relationship between the applied stress $T_{ij}$, and the resulting strain $e_{ij}$, is given by Hooke's law,

$$e_{ij} = \frac{1}{E}[(1 + \nu)T_{ij} - \nu\delta_{ij}T_{kk}]$$

where $E$ is the Young modulus, $\nu$ is the Poisson ratio, and $\delta_{ij}$ is the Kroneker delta.

(a) What is a typical value of the Poisson ratio for rocks.

(b) Suppose an elastic solid undergoes a change in pressure with no shear stress. Relate $e_{11}$ to the change in pressure. (compression is negative)

(c) What happens when $\nu$ approaches a value of 0.5?

(b) Derive the inverse relationship expressing stress in terms of strain.

11 (a) Express the P-wave velocity, $V_p$, in terms of the elastic moduli and the density $\rho$.

(b) Given the relationship in part (a), explain why seismic rays curve upward in the mantle.

(c) Imagine a solid, homogeneous, isotropic planet where the material is perfectly Hookean (i.e., the elastic moduli do not vary with radius). Assume that the planet compresses from self gravitation. Sketch how seismic waves would travel through such a planet.