IGPP

Departmental Examination

1994
Departmental Examination, 1994

This is a 4 hour exam with 12 questions. Write on the pages provided, and continue if necessary onto further sheets. Please identify yourself clearly, and number the pages unambiguously.

You have on average 20 minutes per question, although some will take longer than others. Do not spend too long on any single question! Attempt to answer whatever you can; credit will be given for partial answers.

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1) On the attached map, Figure 1, label 8 plates, 6 spreading ridges, and 4 trenches. (Use names from the following list and add the appropriate type to each name (e.g., plate, ridge, or trench).

<table>
<thead>
<tr>
<th>African</th>
<th>Nazca</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aleutian</td>
<td>North American</td>
</tr>
<tr>
<td>Antarctic</td>
<td>Pacific</td>
</tr>
<tr>
<td>Central Indian</td>
<td>Pacific-Antarctic</td>
</tr>
<tr>
<td>East Pacific</td>
<td>Peru-Chile</td>
</tr>
<tr>
<td>Eurasian</td>
<td>South American</td>
</tr>
<tr>
<td>Indian or Indo-Australian</td>
<td>Southeast Indian</td>
</tr>
<tr>
<td>Java</td>
<td>Southwest Indian</td>
</tr>
<tr>
<td>Mid Atlantic</td>
<td>Tonga-Kermadec</td>
</tr>
</tbody>
</table>

a. Which of the six spreading ridges has the highest spreading rate?
b. Which has the lowest spreading rate?
c. Give a typical value for full spreading rate and provide units (i.e. within the range of observed spreading rates).
d. Choose an example of a ridge, a transform fault, and a trench and sketch on the map the fault-plane solution ("beach ball") you would expect to observe at each one.
e. Given the geometry of the Triple Junction shown in Figure 2, determine the direction and rate of motion between plate A and plate B. What kind of plate boundary is A-B? The boundaries labeled F are transform faults and are slipping at 40 mm/year with right-lateral motion.
2) For the Northridge earthquake (January 17th, 1994) the unit moment tensor is well approximated by

$$\mathbf{M} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. $$

The epicentral coordinate system is $x_1$ North; $x_2$ West; $x_3$ Up. Determine the unit normal vectors and the slip vectors.

What geophysical data would you use to resolve the ambiguity between the fault plane and the auxiliary plane? What is the fault plane for the Northridge earthquake?
3) a. Illustrate with figures why we commonly refer to a layered oceanic crust. What are the geophysical properties and thicknesses of these layers? How do the layers relate to the “geological structure” of oceanic crust?

b. How do the geophysical and geological structures of the crust at fast and slow spreading ridges differ from each other? Illustrate with diagrams if possible.

c. There are structural/morphological and magnetic features that allow the direction and rate of plate motions across an oceanic spreading center to be determined relative to an Euler pole. What are these features and how are they used?

d. An attempt has been made to establish an absolute reference frame for plate motions. What features are commonly used in this context and are they truly fixed relative to each other or the mantle?
4) Explain the meaning of the following terms: (a) geopotential; (b) reference ellipsoid; (c) geoid; (d) geoid height. Give an approximate relationship between geoid height and the geopotential on the reference ellipsoid. Estimate the error in this formula.

One frequently reads about such things as the gravity field of spherical harmonic degree 4, meaning the field that results when only the degree 4 spherical harmonics are summed, while the rest are dropped. Since the spherical harmonic coefficients all vary with the orientation of the reference axes, such functions are seemingly coordinate dependent. Show how to write the degree-$l$ function on the surface of the unit sphere as a single integral of the original function. Hence show the degree-$l$ function does not depend on the choice of axes.
5) Explain what is meant by the term power spectral density (PSD) function. When is it a useful concept?
What techniques are used to make power spectral density estimates consistent? What causes bias in a power spectral density estimate? How can it be reduced?
Sketch the autocorrelation functions and power spectra you would expect to be associated with the time series shown in Figure 3. Don’t forget to label the axes.
At a site well away from the ocean, the ground motion PSD is flat in acceleration from 1 Hz to 10 Hz at a level of \(10^{-14}(\text{m s}^{-2})^2\text{Hz}^{-1}\). What is the root-mean square (RMS) acceleration of the ground in this band? What is the associated RMS ground amplitude?
6) Two well-known statistical estimation techniques are least squares and maximum likelihood estimation. Explain the principle on which each is based and how they differ.

You are given a set of measurements of temperature, $p_i$, at various depths, $x_i, i=1, \cdots, n$, in a borehole. $p$ is believed to increase linearly with depth. Find the least squares estimates of the parameters in such a model and the variance in your parameter estimates. State any assumptions you find it necessary to make. How would you determine whether the linear model is suitable for your data?
7) Give all the evidence you know for the claim that most of the magnetic field observed at the surface of the earth has its sources in a liquid core and that the core has a radius about half the earth's radius. Where does the rest of the field come from and how do we know this?

How is the North magnetic pole defined? How is the virtual geomagnetic pole (VGP) of paleomagnetism defined? What is the equation for the magnetic field of a dipole? Explain briefly how a VGP position is calculated and what observations are needed in this calculation.
8) What is Fermat’s Principle as applied to pulse propagation in acoustics and seismology?

Suppose that a region of the earth is surrounded by boreholes (not necessarily vertically aligned) and the free surface. How would you use Fermat’s principle to conduct a seismic tomography experiment that utilizes both the boreholes and the free surface?
9) A seismic experiment results in the $P$-wave travel time curve shown in Figure 4.

a. Sketch a $P$-velocity vs. depth profile which would result in the observed travel time curve.

b. Sketch a $\tau(p)$ curve for this travel time curve.

c. Indicate on the travel time curve where the largest amplitudes should be expected. Consider a homogeneous half-space with $P$-velocity $\alpha=4$ km/s and $S$-velocity $\beta=2.5$ km/s, and an upgoing SV wave incident upon the free surface.

d. For what range of ray parameters will a reflected $P$-wave not be produced?

A vertically-traveling upgoing $P$-wave with amplitude $A_i$ encounters the velocity vs. depth profile in the near surface shown in Figure 5. The density is constant throughout the profile.

e. What would be the observed amplitude $A_s$ at the surface for (i) the high-frequency limit, and (ii) the low-frequency limit?
10) The constitutive relationship for a perfectly elastic material can be written

$$\tau_{ij} = C_{ijkl} \varepsilon_{kl}$$

where $\tau$ is the stress tensor, $\varepsilon$ is the strain tensor and $C$ is a fourth order tensor of elastic moduli.

a. Why is $\tau$ symmetric?

b. Why is $\varepsilon$ symmetric?

c. Out of a possible 81 coefficients in $C$, how many are independent? (You should make the usual assumption ± what is the usual assumption?)

d. For an isotropic material, how many independent elements of $C$ are there and why?

e. For a transversely isotropic material, how many independent elements of $C$ are there and why?

f. Why are reference Earth models often assumed to be transversely isotropic (at least in solid regions!)?

g. How would you modify the constitutive relationship to include anelasticity?

h. Is it possible for an anelastic solid to exhibit attenuation without physical dispersion? Explain.
11) Use any appropriate numerical method you like to determine which formula (a), (b), (c) goes with which number (A), (B), (C). You need only get a close enough approximation in each case to eliminate two of the three choices ± a two- or three-digit result is quite sufficient. No points for guessing, so give your reasoning.

(a) \[ \sum_{n=2}^{\infty} (-1)^n \ln n \]  

\begin{align*}
\text{Find } df(x) \text{ where } f(x) &= 0 \text{ in} \\
\text{(b)} & \quad f(x) = (x - \cos x)/7 \\
\text{(c)} & \quad \int_{-\infty}^{\infty} 4(64 + x^6) \, dx
\end{align*}

(A) 0.239087

(B) 0.261799

(C) 0.225791
12) Devise a suitable question for this examination that has not been asked and provide an answer for it.