General Notes:

- You have four hours. The exam is closed-book.

- The exam is in two parts. The first contains 10 questions, all of which must be answered. They are brief enough that you should need at most 5 minutes for each (in most instances, much less). You must give an answer for each of these questions.

The second part also contains 10 questions. These are somewhat more difficult, and may require more elaborate answers. You are expected to provide answers to 6 of them, but you may attempt to answer all of them, if you feel that you can do a good job.

- Although it is important to give correct answers, it is at least as important to show that you know what you are doing. You will get little credit for results that appear out of thin air, even if they are correct.

- Remember to exercise good sense, and to show that you have a good physical understanding of the subject. Beware of results which are dimensionally wrong, or answers which are ridiculously off (by many orders of magnitude).

- Please make your answers as readable as you can make them in the time allotted. Brevity is valuable, but not to the point of obscurity. Long rambling essays are not desired either. You should focus on trying to convey your understanding of the questions. If the Committee cannot understand what you have done, they will assume that you do not understand either. Merely jotting down disconnected formulae, without writing what the variables are, does little to show your knowledge.
Section 1

1. The sea surface is an equipotential. What is the point of measuring gravity on board a ship?

2. The matter, magnetization and currents inside the earth produce a geomagnetic field $B$ and a gravitational potential $\phi$. Both these fields can be expanded in spherical harmonics. No proofs are required in what follows.
   a) What is a spherical harmonic? How many linearly independent spherical harmonics are there of degree $n$?
   b) Write the expansions for $B$ and $\phi$ (of course, you need not give the numerical values of the coefficients).
   c) Where are these expansions valid?
   d) Why does the expansion for $B$ have no harmonic term of degree 0?
   e) Why does the expansion for $\phi$ usually have no harmonic terms of degree 1? When might such a term be present?

3. Define “geoid” and “geoid anomaly”. Is there a positive or negative geoid anomaly over a positive mass anomaly? Why?

4. Why is the stress tensor symmetric? What is the most general form of the stress tensor at a free surface with outer unit normal in the positive $x_3$ direction?

5. Give the expressions for P-wave and S-wave velocities in terms of the density $\rho$, the rigidity $\mu$, and the Lamé constant $\lambda$. Why are P-waves faster than S-waves in the outer core?

6. The Indian and Eurasian plates converge at approximately 50 mm/yr in the Himalayan region. Why is there no well-developed “Benioff zone” along the Himalayan segment of the India-Eurasia plate boundary?

7. If a statistical distribution depends on several unknown parameters, what does it mean to say that an estimator of one of these parameters is (mathematical statements are acceptable):
   a) unbiased?
   b) asymptotically unbiased?
   c) consistent?
   d) efficient?
   e) maximum likelihood?

8. Let $x_1, \ldots, x_N$ be independent identically distributed random variables. Suppose we estimate their mean $\mu$ and variance $\sigma^2$ by
\[ \hat{\mu} = \frac{1}{N} \sum_{i=1}^{N} x_i \quad \hat{\sigma}^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \hat{\mu})^2 \]

a) Show that \( \hat{\mu} \) is an unbiased estimator.
b) Show that \( E(x_i x_j) = \sigma^2 \delta_{ij} + \mu^2 \), where \( E \) means expected value and \( \delta_{ij} \) is the Kronecker delta.
c) Show that \( E(x_i \hat{\mu}) = \sigma^2 / N + \mu^2 \).
d) Show that \( E(\hat{\mu}^2) = \sigma^2 / N + \mu^2 \).
e) Show that \( \hat{\sigma}^2 \) is a biased estimator.

9. Prove the following relations between wavenumber \( k \), wavelength \( \lambda \), phase velocity \( c \), group velocity \( U \), frequency \( f \), and period \( T \):

\[
U = c - \frac{\lambda \frac{dc}{d\lambda}}{d\lambda} \quad U = c + k \frac{dc}{dk} \\
U = c^2 \frac{dT}{d\lambda} \quad U = -\lambda^2 \frac{df}{d\lambda}
\]

10. To describe most earthquake mechanisms, we can specify a fault plane with unit normal \( n_i \), a unit slip vector \( d_j \) orthogonal to \( n_i \), and a scalar moment \( M_0 \). Given these, the moment tensor of the equivalent double-couple can be written as \( M_{ij} = M_0 (n_i d_j + n_j d_i) \).

a) Give an expression for the seismic moment. Define your terms and verify dimensions.

b) In a fault mechanism solution (of the “beach ball” type), two nodal planes are determined, the “fault” plane, and the “auxiliary” plane. Find the expression which would give the moment tensor if the earthquake had actually ruptured the “auxiliary” plane instead of the “fault” plane.
c) Give an example of an earthquake source which cannot be represented in terms of a double-couple. Can this happen in the earth?
Section 2

1. Give all the evidence you know for the claim that most of the magnetic field observed at the surface of the earth has its sources in a liquid core whose radius is about half the earth’s radius.

2. The local Gas & Electric company guarantees that all its power is produced in the frequency band 60±0.03 Hz. You want to estimate the power spectrum of the local vertical geomagnetic field Z between periods of roughly one minute and thirty minutes. Your grant contains no money for an analogue attenuator of the 60 Hz signal, so you decide to measure Z every Δ seconds, choosing Δ so that the power-line signal will not be aliased into the lowest third of your Nyquist band. That is, if \( f_N \) is the Nyquist frequency, no power company frequencies alias into frequencies \( f \) satisfying \(-f_N/3 < f < f_N/3\). What is the largest value you can choose for Δ, and if you choose that value, what is the shortest period at which you can estimate the spectrum of Z?

3. The attached figure shows broadband records of the first arrival of an earthquake at Easter Island (27°S, 108°W) recorded at Piñon Flat Observatory (33.6°N, 116.5°W). One component (Z) is the vertical, the other two are geographically oriented (NS and EW) horizontals. Identify the directions of these, and explain your choice. The near-surface P-wave velocity at the station is 6 km/sec; estimate the P velocity at the deepest point of the ray. Is that consistent with what you know of mantle velocities?

4. The NUVEL-1 rigid plate motion model gives the pole of relative motion between the Pacific and North America plates at 48.7°N, 78.2°W, with an angular velocity of 0.78 degrees/Myr.

   a) Calculate the relative plate velocity vector at the location of the Loma Prieta earthquake, (37°N, 121.85°W). Is this consistent with the geodetically estimated rate of slip on the San Andreas of 16 ± 2.5 mm/yr at that location?

   b) In 1906, the San Andreas fault at that location slipped by about 250 ± 60 cm. In 1989, it slipped again by about 150±30 cm. Could you reconcile these observations with the previous results?

   c) Assume that the interseismic deformation (between big earthquakes) across the San Andreas is actually distributed uniformly across a zone extending 40 km on either side of the fault. What is the rate of strain within the fault zone? If you have a measurement system capable of measuring strains of \( 10^{-7} \), how much time do you have to wait in order to have a useful estimate of the rate of slip? Explain your answer.

5. a) It has been proposed that we prevent stress build-up along a major fault like the San Andreas by triggering small (\( M_s = 4 \)) earthquakes in sufficient numbers to accommodate the tectonic slip rate. Presumably, this would eliminate the risk of very large earthquakes (\( M_s = 8 \)). Assuming that we were able to trigger small earthquakes at will, how many magnitude 4 events would we have to endure on a daily basis? Comment on the proposed scheme.
Hints: Assume a magnitude 8 event would happen every 150 years. The triggering technique is an irrelevant mystery. Assume the moment magnitude relation to be $\log_{10} M_0 = 1.5 M_s + 15.5$, where $M_0$ is the moment in dynes cm.

b) Whenever a few naturally occurring magnitude 4 events are felt in California, we invariably get queries from the press as to whether this means the chances of having the BIG one have been thereby reduced. How would you respond to such a query?

Hint: Take the $b$-value of California seismicity to be 1.

6. Sketch the function $Q^{-1}(\omega)$ for an absorption band with a single relaxation peak centered at $\omega_0$. Sketch the associated phase velocity $c(\omega)$, and attenuation coefficient $\alpha(\omega)$, where $k(\omega) = \omega / c(\omega)$ - i $\alpha(\omega)$ is the wavenumber. What is the asymptotic behavior of $Q(\omega)$ at low frequencies $\omega << \omega_0$, and at high frequencies $\omega >> \omega_0$? Is the medium a good propagator of high frequencies, or a bad one? Why don’t we typically see very high frequencies in seismograms? Justify your answers.

7. Sketch a springs-and-dashpots representation of a standard linear solid. Denoting spring constants by $G_1$, $G_2$, etc., and viscous elements by $\eta_1$, $\eta_2$, etc., sketch the form of the stress relaxation and strain retardation curves, and label as many features (e.g. slopes, asymptotes) as you can.

8. The oceanic lithosphere is created during seafloor spreading. At the spreading ridge axis, hot mantle rock (temperature $T_m$) comes in contact with cold seawater (temperature $T_o$) causing it to cool and strengthen. As this young lithosphere is carried away from the spreading axis it continues to cool by diffusion of heat. Use dimensional analysis to arrive at approximate formulas for the variations in surface heat flow and seafloor depth. Don't worry about factors of 2, $\pi$, etc; any nontrivial answer with the proper dimensions is correct. The following parameters will be useful.

(Dimensions M-mass, L-length, T-time):

- $T_m - T_o$: temperature difference across the thermal boundary layer, K
- $\kappa$: thermal diffusivity, $L^2 T^{-1}$
- $t$: age of the lithosphere, T
- $k$: thermal conductivity, $M L T^{-3} K^{-1}$
- $\alpha$: coefficient of thermal expansion, $K^{-1}$
- $\rho_m$: mantle density, $M L^{-3}$
- $\rho_w$: seawater density, $M L^{-3}$

(Temperature, labelled K above for convenience, is actually a dimensionless quantity)
a) What are the dimensions of heat flow? (i.e. $M^i L^j T^k$ where $i, j, k$ are exponents to be supplied). Write the general expression relating heat flow to temperature in the Earth.

b) According to the boundary layer cooling model, the surface heat flow is proportional to $(\pi \kappa t)^{-1/2}$. Add some of the above parameters to this function to develop a more complete formula for the surface heat flow as a function of age. Show that it has the proper dimensions.

c) As the lithosphere cools, it contracts and increases in density causing thermal subsidence. Assuming local isostatic compensation, thermal subsidence is proportional to $(\kappa t)^{1/2}$. Develop a nontrivial and dimensionally correct formula for the seafloor depth as a function of time. How would the rate of thermal subsidence change if $\kappa$ is increased? Why?

d) What other observables are affected by lithospheric cooling?

9. Explain carefully what is meant by the power spectral density (PSD). Under what circumstances is it a useful concept? Describe how the PSD is estimated; how is your method of estimation designed to avoid the pitfalls of the periodogram?

A marine survey in an area of well-developed lineations provides a map of the magnetic anomaly. The one-dimensional power spectral density (PSD) of the anomaly is estimated from profiles normal to the strike of the lineations; the result is shown in the figure (note that both axes are linear, not log).

a) Explain the general features of this figure and give an equation that roughly describes the PSD.

b) What are suitable units for PSD in this case?

c) Supply numerical values for the axes of the figure appropriate for a typical marine situation. Explain how you get these numbers.

d) Sketch the one-dimensional PSD that results from analysis of profiles parallel to the strike.
10. The special creationists (A fundamentalist religious coalition, some of whom have PhD’s in science) claim that the earth is only 10,000 years old. One of their arguments is that the electrical conductivity of the core is estimated (from quantum theory and high pressure experiments) to be about $3 \times 10^5$ mho/meter. The electric currents in a solid sphere with this conductivity and the radius of the earth’s core can maintain only exponentially decaying magnetic fields whose mean lives are shorter than 15,000 years. If the earth were very much older than that, its magnetic field would once have been very much larger than the present field. The rocks contain no evidence for such large paleofields. Discuss this argument as thoroughly and undogmatically as you can (in less than one page!).